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CALCULATION OF IMPEDANCE OF A SHARP PLASMA BOUNDARY WITH A MIXED-TYPE ELECTRON SCATTERING IN ANOMALOUS SKIN-EFFECT CONDITIONS

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Some aspects of calculation of impedance change caused by oblique incidence of electromagnetic wave on a sharp plasma boundary with mixed-type electron scattering in anomalous skin effect (ASE) conditions are considered. Though the corresponding Riemann-Hilbert boundary problem can be reduced [1] to the integral equations of the Fredholm type, formal limit transition to extreme ASE causes kernels singularity. The transformation of the equations to ones with bounded kernels is made by means of partial inversion of the integral operator.

An oblique electromagnetic wave reflection from a sharp plasma boundary with a mixed-type electron scattering can be described with the functions $\Psi_\lambda(k_z)$ and $\Psi_\tau(k_z)$, the linear combinations of the Fourier transforms of electric field components in plasma domain ($z>0$). They should be analytical in the half-plane $\mathrm{Im}(k_z)\leq 0$, together with the functions $\Phi_\lambda(-k_z)$ and $\Phi_\tau(-k_z)$, and should meet two functional equations [2],

$$[1-\Omega^2 q_\lambda(\beta k_{xz})][\Psi_\lambda(k_z)+p\Psi_\lambda(-k_z)]= \Phi_\lambda(k_z),$$
$$[1-k_{xz}^2-\Omega^2 q_\tau(\beta k_{xz})][\Psi_\tau(k_z)-p\Psi_\tau(-k_z)]= \Phi_\tau(k_z), \qquad (1)$$

and the equalities $\Psi_\lambda(\pm i k_x)=\pm i\Psi_\tau(\pm i k_x)$. Here $\beta=v_F\omega[c(\omega+i\nu)]^{-1}$, $\Omega=\omega_e[\omega(\omega+i\nu)]^{-1/2}$, $k_{xz}=(k_x^2+k_z^2)^{1/2}$, c is the speed of light, ω is the frequency, ω_e is the plasma frequency, ν is the collision frequency, v_F is the velocity of electrons at the Fermi surface (spherical), k_x is dimensionless (in units ω/c) transverse wave number (sine of wave incidence angle), $p\in(0,1)$ is the fraction of electrons with specular boundary scattering, $q_\lambda(\chi)=3\chi^{-2}\{(2\chi)^{-1}\ln[(1+\chi)/(1-\chi)]-1\}$, $q_\tau(\chi) =3(2\chi^2)^{-1}\{1-(2\chi)^{-1}(1-\chi^2)\ln[(1+\chi)/(1-\chi)]\}$. The problem is considered under the approximation $\beta<<1$, $A>>1$, where $A=(\beta\Omega)^{2/3}$, so, the distance traveled by an electron during the oscillation period is large compared with the effective field penetration depth.

Introducing new variables and functions, one can write (1) in the form

$$X_\lambda(u)+pQ_\lambda^\times(u)X_\lambda(-u)=Y_\lambda(-u), \qquad X_\tau(u) -pQ_\tau^\times(u)X_\tau(-u)=Y_\tau(-u), \qquad (2)$$

where $X_\lambda(u)= \Psi_\lambda(0)^{-1}\Psi_\lambda((i\beta u)^{-1})Q_\lambda^+(u)$, $X_\tau(u)=\Psi_\tau(-c_\tau u)Q_\tau^+(u)$, $c_\tau=-ic_3 A\beta^{-1}$, $c_3=\exp(i\pi/3)$. Here and further, a function with the upper index + should be analytic in the half-plane $\mathrm{Im}(u)>0$, and it should be built for the corresponding function without upper index in accordance with the identity $Q^+(u)Q^+(-u)=Q(u)$. A corresponding function with the upper index × is determined by the identity $Q^\times(u)=Q^+(u)[Q^+(-u)]^{-1}$. The functions $Q_\lambda(u)$ and $Q_\tau(u)$ correspond to the first factors in the (1) left hand sides multiplied with the constants. The identity for $Q^+(u)$ remains its sign indefinite, and therefore the signs are chosen to provide $Q_\lambda^+(i\infty)=1$ and $\lim_{u\to i\infty}[Q_\tau^+(u)/u]=c_\tau$.

By using (here and further) the tilde symbol for the Fourier transforms over all real u axes, from the first of (2) one can obtain the following equation:

$$\tilde{X}_\lambda(\zeta) + p\int_0^\infty d\xi \tilde{Q}_\lambda^x(\zeta + \xi)\tilde{X}_\lambda(\xi) + p\tilde{Q}_\lambda^x(\zeta) = 0 \quad (\zeta > 0), \quad (3)$$

(3) can be solved by iterations [3], and some values, in particular, the quantity $F_\lambda = \lim_{u\to i\infty}\{iu[Q_\lambda^+(u) - X_\lambda(u)]\}$, can be efficiently calculated. The function $X_\tau(u)$ can be written as linear combination of the functions $X(\pm p; u)$ determined by the equations

$$X(p'; u) - p'Q_\tau^x(u)X(p'; -u) = Y(p'; -u) \quad (4)$$

(for $p' = p$ and for $p' = -p$), and with the requirements that the functions $X(p'; u)$ and $Y(p'; u)$ are analytic and bounded in the half-plane $\text{Im}(u) \geq 0$, and the condition $X(p'; i\infty) = 1$ is satisfied. From (4), an equation similar to (3) can be obtained and the quantity $\Psi_{\tau 1} = i\lim_{u\to i\infty}[uX(-p; u) - c_\tau^{-1}Q_\tau^+(u)]$ can be calculated, however to calculate some other quantities one has to transform the equation. Taking $a > 0$, for the Fourier transforms of the functions $x(a, p'; u) = X(p'; u)(ia + u)^{-1}$, $g(a; u) = Q_\tau^x(u)(ia - u)(ia + u)^{-1}$, $h(a, p'; u) = Y(p'; ia)(ia + u)^{-1}$, one can obtain an integral equation:

$$\tilde{x}(a, p'; \zeta) = p'\int_0^\infty d\xi \tilde{g}(a; \zeta + \xi)\tilde{x}(a, p'; \xi) + \tilde{h}(a, p'; \zeta) \quad (\zeta > 0). \quad (5)$$

The function $|\tilde{g}(a; \zeta)|$ for $1 \ll \zeta \ll A$ is close to $(\pi\zeta)^{-1}$, so, in the limit of $A \to \infty$, (5) is not a Fredholm-type equation. For $p' = p$ and bounded A, one can take $a = 1$ and write the equation $\tilde{x}(1, p; \zeta) = p\int_1^A d\xi[\pi(\zeta + \xi)]^{-1}\tilde{x}(1, p; \xi) + \tilde{H}(\zeta)$, where the term $\tilde{H}(\zeta)$ involves the integrals with the unknown function $\tilde{x}(1, p; \zeta)$ and kernels bounded with some functions $([(\zeta+\xi)^{-2}\ln(\zeta+\xi) + A^{-2}(\zeta+\xi)]$ for $\{\zeta \in (1, A), \xi \in (1, A)\}$, $A(\zeta+\xi)^{-2}$ for $\max(\zeta, \xi) > A$, and $\max(1, \zeta, \xi)^{-1}$ for other domains. In brackets, here and further the factors independent on ζ, ξ, and A are omitted. Considering the function $\tilde{H}(\zeta)$ as known, one can build the solution of the last equation applying the second of two methods used in [4]. The kernel of the corresponding regularization operator is bounded with the product $\min(\zeta, \xi)^{-\kappa_p}\max(\zeta, \xi)^{\kappa_p - 1}$ (where $\kappa_p = \pi^{-1}\arcsin(p)$), and the kernel of the obtained equation, for $\zeta \in (1, A)$, is bounded with the functions $\zeta^{\kappa_p - 1}$, $\zeta^{\kappa_p - 1}\xi^{-1} + A^{\kappa_p - 1}\zeta^{-\kappa_p}$, and $A^{1+\kappa_p}\zeta^{-\kappa_p}\xi^{-2}$ in intervals $\xi \in (0, 1)$, $\xi \in (1, A)$, and $\xi > A$, respectively. The introduction of the variable $\overline{\zeta}$ and the function $\overline{x}(\zeta)$, so that the products of derivative $d\overline{\zeta}/d\zeta$ with some factors (1 for $\zeta \in (0, 1)$, $\zeta^{2-\kappa_p}$ for $\zeta \in (1, A^{\kappa_p})$, $A(\zeta / A)^{1-\kappa_p}$ for $\zeta \in (A^{\kappa_p}, A)$, ζ^2 / A for $\zeta > A$) leads to the bounded limits for fixed ζ if $A \to \infty$. The value $\overline{x}(\zeta)/\tilde{x}(1, p; \zeta)$ is equal to 1, $\zeta^{1-\kappa_p}$, and $\zeta A^{-\kappa_p}$ in the intervals $\zeta \in (0, 1)$, $\zeta \in (1, A)$, and $\zeta > A$, respectively. All this yields an equation with the kernel bounded even for $A \to \infty$.

In the case $p' = -p$, to keep the sign in the estimation of the product $p'\tilde{g}(a; \zeta)$ for $1 \ll \zeta \ll A$, it is reasonable to take positive $\delta_1 \sim A^{-1}$ and consider (5) for $a = \delta_1$. Denoting

$c_\sigma = c_3{}^2 A^{-1}$, $\delta_0 = -\mathfrak{i} c_\sigma{}^{1/2}$, $C_1 = -\mathfrak{i} X(p; \mathfrak{i}\delta_0) \delta_0^{\kappa_p}$, $C_0 = -\mathfrak{i} X(-p; \mathfrak{i}\delta_0) \delta_0^{-\kappa_p}$, and using the Euler gamma-function, for $1 << \zeta << A$, one can obtain $\tilde{x}(1, p; \zeta) \Gamma(\kappa_p) \approx C_1 \zeta^{\kappa_p - 1}$, $\tilde{x}(\delta_1, -p; \zeta) \Gamma(1 - \kappa_p) \approx C_0 \zeta^{-\kappa_p}$. In the limit of A→∞, the quantities C_1 and C_0 tend to finite real values, which can be effectively calculated from the solutions of the equations with bounded kernels obtained from (5).

Suppose that the functions $X_\sigma(p'; u)$ and $Y_\sigma(p'; u)$ are analytic and bounded in the half-plane $Im(u) > 0$ and obey the functional equation

$$X_\sigma(p'; u) - p' Q_\tau{}^x (c_\sigma / u) X_\sigma(p'; -u) = Y_\sigma(p'; -u) \qquad (6)$$

and the condition $X_\sigma(p'; \mathfrak{i}\infty) = 1$. Then the identity $X(p'; u) = X(p'; 0) X_\sigma(p'; c_\sigma / u)$ takes place. Applying the limit transition to the explicit solution of the equation, in which the function $Q_\tau{}^x(u)$ is approximated with the relation of the polynomials, one can obtain the equality $X(p; 0) X(-p; 0) = 1$. From (6), an equation similar to (3) can be obtained and the quantity $F_\sigma = \mathfrak{i} \lim_{u \to \mathfrak{i}\infty} \{ u^2 (\partial / \partial u) \ln[X_\sigma(-p; u) / Q_\tau{}^+ (c_\sigma / u)] \}$ can be calculated. Besides, (6) can be treated similarly to (4), and can equation similar to (5), but with another function instead of $\tilde{g}(a; \zeta + \xi)$, may be obtained. For the quantities $C_{\sigma 1} = -\mathfrak{i} X_\sigma(-p; \mathfrak{i}\delta_0) \delta_0^{\kappa_p}$, $C_{\sigma 0} = -\mathfrak{i} X_\sigma(p; \mathfrak{i}\delta_0) \delta_0^{-\kappa_p}$ (which one can effectively calculate considering now the cases $p' = -p$, $a = 1$ and $p' = p$, $a = \delta_1$ and evaluating the solutions of the corresponding integral equations for large ζ), and for the quantity $F_\tau = X(-p, 0)(c_3 A)^{\kappa_p}$, the identities $C_0 C_{\sigma 1}{}^{-1} = F_\tau = C_{\sigma 0} C_1{}^{-1}$ take place.

Under the assumption $\{A >> 1, \beta << 1\}$, for dimensionless impedance Z one can obtain approximate identity $\beta(AZ)^{-1} \approx \Psi_{\tau 1} \exp(\mathfrak{i}\pi/3) + k_x{}^2 \beta^2 A^{-1-2\kappa_p} \exp(-2\mathfrak{i}\pi\kappa_p/3) F_Z$, in which the quantities $\Psi_{\tau 1}$ and $F_Z = (F_\lambda - F_\sigma) F_\tau{}^2$ in the limit $\{\beta \to 0, A \to \infty\}$ turn positive. The complex factors in the expression for the impedance are obtained explicitly due to introduction of convenient variables, for which the corresponding functions in the functional equations in the extreme ASE limit become positive for real values of arguments.

The quantities C_1, C_0, and $\Psi_{\tau 1}$ can also be calculated with the method of [5].

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